SIDDHARTH INSTITUTE OF ENGINEERING &TECHNOLOGY:: PUTTUR
DEPARTMENT OF MATHEMATICS

DISCRETE MATHEMATICS (18HS0836)

QUESTION BANK

UNIT-1
Mathematical Logic

1. a) Define statement . [2M]

b) Define tautology with examples. [2M]

c) Write the following statement in symbolic form, If either Jerry takes calculus or Ken takes sociology, then Larry will take English. [2M]

d) Write the negation of the statement "Today is Friday" and express this in simple English.[2M]

e) Write dual to the $p \rightarrow (q \wedge r)$ [2M]

2. a) Construct the truth table for the following formula $\neg(\neg P \vee \neg Q)$ [5M]

b) Construct the truth table to Show that $\neg P \wedge (Q \wedge P)$ is a contradiction. [5M]

3. a)Define NAND,NOR & XOR and give their truth tables. [5M]

b) Show that the following set of premises are inconsistent.

$P \rightarrow Q, P \rightarrow R, Q \rightarrow \sim R, P.$ [5M]

4. a)Show that $S \vee R$ is a tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$ [5M]

b) Show that $(P \vee Q) \rightarrow R \equiv (P \rightarrow R) \wedge (Q \rightarrow R)$ [5M]

5. a) Obtain the disjunctive normal form

$P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$ [5M]

b)Obtain the principle conjunctive normal form $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$ [5M]

6. a)Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$ [5M]

b)Show that $(\forall x)(P(x) \rightarrow Q(x)) \wedge (\forall x)(Q(x) \rightarrow R(x)) \Rightarrow (\forall x)(P(x) \rightarrow R(x))$ [5M]

7. a)Define Quantifiers and types of Quantifiers with examples. [5M]

b)Show that $(\exists x)\ M(x)$ follows logically from the premises

$(\forall x)(H(x) \rightarrow M(x))$ *and* $(\exists x)H(x)$ [5M]

8. a) Use indirect method of proof to prove that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$ [5M]

b) Define Maxterms&Minterms of P & Q & give their truth tables [5M]

9. a) Show that $\sim P$ is a valid conclusion from the premises

$\sim(P \wedge \sim Q)$, $\sim Q \vee R$, $\sim R$ [5M]

b) Obtain PCNF of $A = (p \wedge q) \vee (\neg p \wedge q) \vee (q \wedge r)$ by constructing PDNF [5M]

10. a) Show that $P \vee Q$ follows form P [5M]

b) Show that $\overset{s}{\Rightarrow}(\neg Q \wedge (P \to Q)) \to \neg P$ [5M]

UNIT-2

Relations & Algebraic Structures

1. a) Define Poset and Hasse diagram. [2M]

b) Let X = {1,2,3,4} and R = {(x,y) / x > y} give its Matrix form. [2M]

c) If $f : R \to R \ni f(x) = \frac{2x+3}{5}$, *find* $f^{-1}(x)$ [2M]

d) Define semi group with example. [2M]

e) Define homomorphism. [2M]

2. a) If R be a relation in the set of integers Z defined by $R = \{(x,y) : x \in Z, y \in Z, (x-y) \text{ is divisible by } 6\}$ then prove that R is an equivalence relation. [5M]

b) Let A= { 1,2,3,4,5,6,7 } .determine a relation R on A by $aRb \Leftrightarrow 3\ divides(a-b)$, show that R isan equivalence relation ? [5M]

3. Let A = { 1,2,3,4}and and let R ={ (1,1),(1,2) ,(2,1),(2,2), (3,4), (4,3), (3,3) , (4,4)} be an equivalence relation on R ? determine A/R. [10M]

4. Let A be a given finite set and P(A) its power set . let $\subseteq$ be the inclusion relation on the elements of P(A) .Draw the Hass diagram of (P(A) , $\subseteq$) for i) A = { a } ii) A ={ a ,b} iii) A = { a,b,c } iv) A = { a,b,c,d }. [10M]

5.a) Define a binary relation. Give an example.Let R be the relation from the set A = {1, 3, 4} on itself and defined by R = { (1, 1), (1, 3), (3, 3), (4, 4)} the find the matrix of R draw the graphof R . [5M]

b) verify $f(x) = 2x + 1, g(x) = x$ for all $x \in R$ are bijective from $R \to R$ [5M]

6.a) Let $f: A \to B$, $g: B \to C$, $h: C \to D$ then prove that $ho(gof) = (hog)of$ [5M]

b) If $f: R \to R$ such that $f(x,y) = 2x+1$ *and* $g : R \to R$ *such that* $g(x) = \frac{x}{3}$ then verify that$(gof)^{-1} = f^{-1}og^{-1}$ [5M]

7.a) Prove that the set Z of all integers with the binary operation *, defined as $a*b = a+b+1, \forall a,b \in Z$ is an abelian group. [5M]

b)Define and give an examples for group, semigroup, subgroup &abelian group. [5M]

8.a) Let $S = \{a,b,c\}$ and let * denotes a binary operation on 'S' is given below also let $P = \{1,2,3\}$ and addition be a binary operation on 'p' is given below. show that $(S,*)$ & $(P,\oplus)$ are isomorphic.[5M]

(+)	1	2	3
1	1	2	1
2	1	2	2
3	1	2	3

*	A	B	C
A	A	B	C
B	B	B	C
C	C	B	C

b) On the set Q of all rational number operation * is defined by $a*b = a+b-ab$

Show that this operation Q froms a commutative monoid. [5M]

9. a) Show that the set{1,2,3,4,5} is not a group under addition and multiplication modulo 6.[5M]

b) Show that the binary operation * defined on $(R,*)$ where $x * y = x^y$ is not associative. [5M]

10.a)show that the set of all roots of the equation $x^4 = 1$ forms a group under multiplication. [5M]

b)Explain the concepts of homomorphism and isomorphism of groups with examples [5M]

UNIT-3
Elementary Combinatorics

1. a)In howmany ways 5 students can be selected from 12 students without student. [2M]
   b) How many different words can be formed with the letters of the word MISSISSIPPI. [2M]
   c) Evaluate $\binom{12}{5,3,2,2}$ [2M]
   d) Find the coefficient of x^3y^4 in the expansion of $(x+y)^7$ [2M]
   e) State Binomial Theorem. [2M]
2. a) Enumerate the number of non negative integral solutions to the inequlity $x_1+x_2+x_3+x_4+x_5 \le 19$. [5M]
   b) Howmany integral solutions are there to $x_1+x_2+x_3+x_4+x_5=20$ where each(i) $x_i \ge 2$ (ii) $x_i > 2$ [5M]
3. a) How many numbers can be formed using the digits 1, 3, 4, 5, 6, 8 and 9 if no repetitions [5M] areallowed?
   b) What is the co-efficient of (i) x^3y^7 in $(x+y)^{10}$ (ii) x^2y^4 $in(x-2y)^6$
4. a) Out of 5 men and 2 women, a committee of 3 is to be formed. In how many ways [5M]
   Can it be formed if at least one woman is to be included? [5M]
5. a) The question of mathematics contains two questions divided into two groups of 5 questions [5M] each.In howmany ways can an examine answer six questions taking atleast two questions from each group.

   b) How many permutations can be formed out of the letters of word “SUNDAY”? How many of these [5M]
   (i) Begin with S? (ii) End with Y? (iii) Begin with S & end with Y? (iv) S &Y always together?
6. a)Inhowmany ways can the letters of the word COMPUTER be arranged? Howmany of them [5M] begin with C and end with R? howmany of them do not begin with C but end with R?

   b) Outof 9 girls and 15 boys howmany different committees can be formed each consisting of 6 [5M] boys and 4 girls?
7. a) Find the coefficient of (i) $x^3y^2z^2$ $in(2x-y+z)^9$ (ii) x^6y^3 $in(x-3y)^9$ x^6y^3 [5M]
   b) Find how many integers between 1 and 60 that are divisible by 2 nor by 3 and nor by 5. [5M] Alsodetermine the number of integers divisible by 5 not by 2, not by 3.
8. a) Out of 80 students in a class, 60 play foot ball, 53 play hockey and 35 both the games. [5M] How many students (i) do not play of these games? (ii) Play only hockey but not foot ball
   b) A Survey among 100 students shows that of the three ice cream flavours vanilla,chocolate, [5M] strawberry.50 students like vanilla,43 like chocolate,28 like straw berry,13 like vanilla and chocolate11 like chocolate and straw berry,12 like straw berry and vanilla and 5 like all of them. Find the following.
   1. Chocolate but not straw berry
   2. Chocolate and straw berry but not vanilla
   3. Vanilla or chocolate but not straw berry
9. a)How many different license plates are there that involve 1,2or 3 letters followed by 4 digits ? [5M]
   b)Find the minimum number of students in a class to be sure that 4 out of them are born on the same month.? [5M]
10. a)Applying pigeon hole principle show that of any 14 integers are selected from the set [5M] S = {1, 2, 3… 25 } there are atleast two whose seem is 26. Also write a statement that generalizes this result
    b)Show that if 8 people are in a room, at least two of them have birthdays that occur on the same day of the week. [5M]

UNIT-4
Recurrence Relation

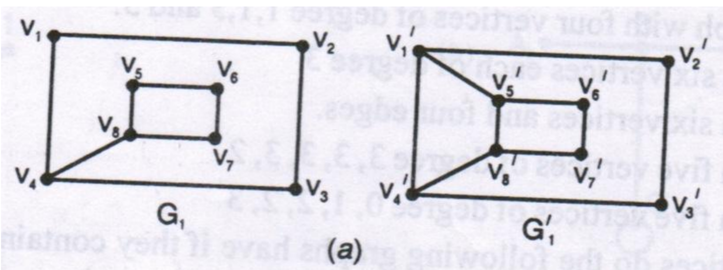
1. a) State generating function. [2M]
   b)Determine the coefficients of x^{15} $in\, x^3(1-2x)^{10}$ [2M]
   c) Define Recurrence relation. [2M]
   d) Find order of recurrence relation $a_{n+2}-a_{n+1}=2a_n$ [2M]
   e) Find the generating function for the sequence 1, 2, 3, 4… [2M]
2. a)Determine the sequence generated by (i) $f(x)=2e^x+3x^2$ (ii) $f(x)=e^{8x}-4e^{3x}$. [5M]
   b) Find the sequence generated by the following generating functions
   (i) $(2x-3)^3$ (ii) $\dfrac{x^4}{1-x}$ [5M]
3. a)Solve $a_n=a_{n-1}+2a_{n-2}, n>2$ with condition the initial $a_0=0, a_1=1$ [5M]
   b) Solve $a_{n+2}-5a_{n+1}+6a_n=2$ with condition the initial $a_0=1, a_1=-1$ [5M]
4. a)Solve the R.R $a_{n+2}-2a_{n+1}+a_n=2^n$ with initial condition $a_0=2, a_1=1$ [5M]
   b) Using generating function solve $a_n=3a_{n+1}+2$, $a_0=1$ [5M]
5. a)**S**olvethe following $y_{n+2}-y_{n+1}-2y_n=n^2$ [5M]
   b) Solve $a_n-5a_{n-1}+6a_{n-2}=1$ [5M]
6. a)Solve the recurrence relation $a_r=a_{r-1}+a_{r-2}$ Using generating function. [5M]
   b)Solve the recurrence relation using generating functions $a_n-9a_{n-1}+20a_{n-2}=0$ for $n\geq 2$
   and $a_0=-3, a_1=-10$ [5M]
7. a)Solve the recurrence relation $a_n=a_{n-1}+\dfrac{n(n+1)}{2}$ [5M]
   b) Solve $a_k=k(a_{k-1})^2$, $k\geq 1, a_0=1$ [5M]
8. a) $a_n=2a_{n-1}-a_{n-2}$ with initial conditions $a_1=1.5$ & $a_2=3$ [5M]
   b) $a_n=3a_{n-1}-a_{n-2}$ with initial conditions $a_1=-2$ & $a_2=4$ [5M]
9. a)Solve $a_n-7a_{n-1}+10a_{n-2}=4^n$ [5M]
   b) Solve $a_n=a_{n-1}+2a_{n-2}$ $n>2$ with condition the initial $a_0=2, a_1=1$ [5M]
10. a)Solve $a_n-5a_{n-1}+6a_{n-2}=2^n$, $n>2$ with condition the initial $a_0=1, a_1=1$.
    Using generating functions. [5M]
    b) Solve $a_n-4a_{n-1}+4a_{n-2}=(n+1)^2$ given $a_0=0, a_1=1$ [5M]

UNIT-5
Graph Theory

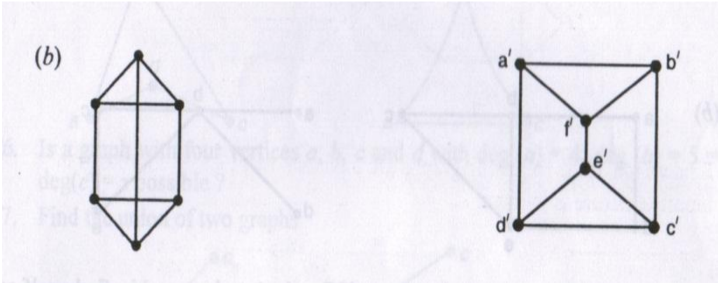
1. a) Define Path. [2M]
   b) Define complete graph. [2M]
   c) State Eulers formula. [2M]
   d) State handshaking theorem. [2M]
   e) Define Spanning Tree. [2M]

2. a)Explain In degree and out degree of graph. Also explain about the adjacency matrix representation of graphs. Illustrate with an example? [5M]
   b)Give an example of a graph that has neither an Eulerian circuit nor a Hamiltonian circuit. [5M]
3. a)Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$
   b)Explain about complete graph and planar graph with an example [5M]
4. a)A graph G has 21 edges, 3 vertices of degree4 and the other vertices are of degree 3.Find the number of vertices in G? [5M]
   b)Define isomorphism. Explain Isomorphism of graphs with a suitable example [5M].
5. a)Suppose a graph has vertices of degree 0 , 2, 2, 3 and 9 . How many edges does thegraph have ? [5M]
   b)Give an example of a graph which is Hamiltonian but not Eulerian and vice versa[5M]
6. a)Let G be a 4 – Regular connected planar graph having 16 edges. Find the number of regions of G. [5M]
   b)Draw the graph represented by given Adjacency matrix [5M]

   (i) $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$
7. a)Show that in any graph the number of odd degree vertices is even. [5M]
   b)Identify whether the following pairs of graphs are isomorphic or not? [5M]



8. a)Explain about the Rooted tree with an example ? [5M]
   b)Show that the two graphs shown below are isomorphic ? [5M]



9. a)Find the chromatic polynomial & chromatic number for $K_{3,3}$ [5M]
   b)Explain graphcoloring and chromatic number give an example [5M]
10. Explain Depth- First-Search, Breadth-First-Search Algorithm [10M]